BELLCOMM, INC.

SUBJECT: Calculating Dynamic Soil Bearing Strength for Vertical Landing Spacecraft - Case 340

DATE: December 7, 1966

FROM: P. L. Chandeysson

ABSTRACT

An analysis of the dynamic bearing strength of soils being penetrated by the footpads of a vertical landing spacecraft is described. Dynamic bearing strength formulas are derived for soils which fail either by compression or by sidewise displacement. These results are generalized into a dynamic bearing strength formula which is applicable to soils which fail in either one or a combination of these two modes.

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MEMORANDUM FOR FILE

INTRODUCTION

The dynamic bearing strength of a soil being penetrated by a solid object such as a footpad of a landing spacecraft is usually considered to be greater than, or at least different from the static bearing strength. To calculate the soil reactions on the footpad, the static bearing strength can be considered to be augmented by a series of empirical dynamic terms proportional to various powers of the penetration velocity and acceleration. A term involving the square of the penetration velocity is a favorite in this empirical approach because the drag on solid bodies penetrating fluids is proportional to velocity squared. Since each additional term introduces a new independent variable into the problem, the number of such terms is limited by practical considerations. The empirical coefficients must be evaluated for each type of soil, and this requires a great amount of experimental effort using elaborate equipment. Full scale equipment must be used unless scaling laws are established. The effect of reduced gravitational acceleration is difficult to simulate in tests.

Soil mechanics in general is not amenable to very accurate analysis; the static bearing strength of soil cannot be reliably calculated within a factor of two from the engineering parameters of the soil. However, since the limitations of a purely empirical approach to dynamic bearing strength are well recognized, attempts have been made to develop approximate analysis to indicate which of the dynamic terms are needed and to relate the coefficients to the soil properties.

ANALYSIS OF DYNAMIC BEARING STRENGTH

Professor Ronald F. Scott of the California Institute of Technology has devised an analysis of dynamic bearing strength useful for calculating the reaction on objects penetrating homogeneous soils at low velocities* normal to the surface.(1) In this analysis, the

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*Low velocities are those at which heating caused by dissipation of kinetic energy has a negligible effect on the soil properties. They should also be less than the sonic velocity in the soil.

dynamic bearing strength of the soil is the sum of the static bearing strength and a dynamic augmentation pressure resulting from the dynamic forces needed to move the soil out of the way of the penetrating object. Calculating these dynamic forces requires knowledge of how the soil moves in response to the penetrating object. The soil motion during dynamic penetration is considered to be essentially the same as that produced by static penetration or settling. Since the motion of soil under settling loads has long been studied as a means of calculating static bearing strengths, this analysis makes available a means of calculating dynamic bearing strengths of soils based on their known static behavior.

The first proposition, that the dynamic bearing strength is the sum of the static bearing strength and some velocity dependent terms, has been accepted by those who use empirical terms to calculate dynamic bearing strength. It seems reasonable to assume that the static or velocity independent terms of dynamic bearing strength are the same as the static bearing strength if, as the second proposition states, the soil actually deforms during dynamic penetration in the same way as it deforms during a static loading. However, this is not always completely true.

For example, one can visualize the impact of a baseball thrown down into dry sand. The shower of sand grains clearly indicates a different soil motion than would occur if the baseball were slowly pushed into the sand. The theory is not perfectly applicable to this situation because some of the soil particles have achieved enough velocity to fly up in the air rather than to be pushed up in a ring shaped mound. However, the theory will be approximately correct if most of the soil particles move in the same way they would have moved during a static penetration. Moreover, an examination of the soil velocities predicted during impact may indicate how the soil will depart from the static flow patterns, thereby enabling a corrected soil flow model to be devised. In the absence of accurate dynamic soil flow models, the static flow model provides at least a starting point from which a rational means of calculating the dynamic bearing strength can be developed.

Professor Scott has proposed two different soil flow models based on the static behavior of compressible and incompressible soils. The compressible soil model is simpler and will be described first.

DYNAMIC BEARING STRENGTH OF COMPRESSIBLE SOIL

As the compressive stress is increased in a static loading test, certain types of cohesive soil and volcanic rocks undergo a compressive type failure involving a density change. The material, originally at density ρ_1, crushes to density ρ_2 at a critical value of normal stress σ_c. The relative density change is described by the soil compressibility, C.

$$C = \frac{\rho_2 - \rho_1}{\rho_2} \quad (1)$$

Compressibility is bounded between zero and one, being zero for an incompressible soil and one for a soil which compresses down to zero volume.

When an object such as the footpad of a spacecraft penetrates a compressible soil, a slug of compressed soil is formed below the footpad, Figure 1. This slug of soil moves downward at the same speed as the footpad, growing in length as the footpad penetrates. The soil crushes from density ρ_1 to ρ_2 at the lower end of the slug of soil at a depth, z, below the surface. The mass of this slug of soil per unit area of footpad, A, is:

$$\frac{M}{A} = \rho_1 z = \rho_2 (z - y) \quad (2)$$

where y is the depth of penetration of the footpad. From this it follows that

$$z = \frac{Y}{C} \quad (3)$$

and

$$\frac{M}{A} = \frac{\rho_1}{C} y \quad (4)$$

Under static conditions an effective pressure, p, must be exerted at the crush depth, z, in order to propagate the crushing interface farther into the soil. This effective crush pressure includes not only the normal crushing stress, σ_c, but also accounts for the shear and friction forces acting on the periphery of the slug of soil. Effective crush pressure varies approximately linearly with crush depth for depths of the order of one footpad diameter.(1)

$$p = p_o + bz \quad (5)$$

The static bearing strength, P_s, of this soil is the effective crush pressure of the soil minus the weight of the soil slug above the crush interface:

$$P_s = P_o + bz - g\left(\frac{\rho_1}{C} y\right)$$

or,

$$P_s = P_o + \left(\frac{b - \rho_1 g}{C}\right) y \tag{6}$$

where g is the local acceleration of gravity. Thus the static bearing strength of compressible soil varies linearly with depth, either increasing or decreasing with depth depending on the relative magnitudes of b and $\rho_1 g$. If these quantities are equal, the soil static bearing strength is independent of depth.

The four engineering parameters ρ_1, C, p_o and b characterize compressible soil.

To calculate the dynamic bearing strength, the dynamic augmentation should be added to the static bearing strength. The dynamic augmentation is the additional force per unit area of footpad needed to move the soil slug at penetration velocity. The dynamic force is equal to the time rate of change of momentum of the soil slug moving with the footpad. This slug of soil has the same velocity, $\dot{y}$, and acceleration, $\ddot{y}$, as the footpad and a mass per unit area of $\frac{\rho_1}{C} y$. The rate of change of momentum per unit area is the dynamic bearing strength augmentation, P_A.

$$P_A = \frac{d}{dt}\left(\frac{M}{A}\dot{y}\right) = \frac{\rho_1}{C}\dot{y}^2 + \frac{\rho_1}{C} y\ddot{y} \tag{7}$$

The total dynamic bearing strength, P_{DYN}, of the compressible soil is the sum of the static bearing strength and the dynamic augmentation.

$$P_{DYN} = p_o + \left(\frac{b - \rho_1 g}{C}\right) y + \frac{\rho_1}{C}\dot{y}^2 + \frac{\rho_1}{C} y\ddot{y} \tag{8}$$

DYNAMIC BEARING STRENGTH OF INCOMPRESSIBLE SOIL

Incompressible soil can bear an unlimited normal stress without a crushing type failure; therefore, the density, ρ, is constant. This material fails in shear and the maximum shear stress, T_y, generally depends on the normal stress, σ.

$$\tau_y = c + \sigma \tan \phi$$

The cohesive shear strength, c, is increased by a frictional or normal stress dependent shear strength term. The angle ϕ is known as the angle of internal friction and ranges from 0° to 40° for normal earth soils. Some soils such as dry sand have zero cohesive shear strength; most soils have both cohesive and frictional shear strength.

The three engineering parameters ρ, c and ϕ characterize incompressible soil.

When an object, considered to be a round flat footpad of radius r, slowly penetrates an incompressible soil, the soil is displaced radially and upward around the footpad. Several volumes of moving soil are formed as shown in Figure 2. A conical volume of soil, #1, is trapped beneath the footpad and moves downward at the same speed as the footpad. Soil moves radially and upward through ring shaped volumes, #2 and #3, with velocities somewhat less than the footpad velocity because of the increased flow area. The displaced soil moves upward through volume #4, forming a ring shaped mound protruding above the original surface. Since the soil is incompressible, the volume of this mound must equal the volume of displaced soil.

The geometry of the static soil flow model depends on the angle of internal friction of the soil.[2] For soils with zero angle of internal friction, the flow geometry is as shown in Figure 2. The tip angle of the conical soil volume #1 is 90°, the curved boundaries of volume #2 are circular arcs, and the width of the displaced ring of soil is about twice the footpad radius. For frictional soils, the flow geometry changes. The tip angle of soil volume #1 is reduced from 90° by the angle of internal friction, the curved boundaries of volume #2 become logarithmic spirals, and the entire failing region spreads out. Although this changes the mass and flow velocity within the soil volumes, the values do not differ greatly from those of a frictionless soil[1]; hence, the soil flow model for a frictionless soil has been adopted for all incompressible soils for the purpose of calculating the dynamic effects. The dynamic augmentation terms will therefore be independent of ϕ. Including the effect of friction on the dynamic terms is a possible refinement.

Referring to the geometry of Figure 2 and considering the soil as continuous and incompressible, the soil mass and average soil velocity of each of the four soil volumes can be calculated in terms of the soil density, ρ, the footpad radius, r, the penetration depth, y, and the penetration velocity, $\dot{y}$. These quantities will be needed later to calculate the dynamic augmentation and are shown in Table I. Only volume 4 has a soil mass which changes with penetration depth. Since the soil is incompressible, the velocity is inversely dependent on the flow area. The flow area in a soil volume is considered as the average of the entrance and exit flow areas. For example, the flow area for soil entering volume #2 is the conical interface between volumes #1 and #2 and equals $\sqrt{2}\,\pi r^2$. The exit flow area is the interface between volumes #2 and #3 and equals $3\sqrt{2}\,\pi r^2$. The average flow area in volume #2 is taken as $2\sqrt{2}\,\pi r^2$, and the average flow velocity is $\dot{y}/2\sqrt{2}$ or $\dot{y}/2.83$.

The static bearing strength of incompressible soils is of considerable engineering interest and is discussed in textbooks concerned with the design of foundations for civil engineering structures. A formula suggested for calculating the bearing strength of soils under circular footings can be used to find the static bearing strength, P_s, for incompressible soil.[3]

$$P_s = 0.6\ \rho g r N_\gamma + \rho g y N_q + 1.3 c N_c \qquad (10)$$

This formula is based on an analytical model which considers a round flat footpad acting on the level surface of a homogeneous, semi-infinite soil mass. The effects of internal friction are considered in this model since friction has an important effect on the static bearing strength of incompressible soils. The bearing capacity factors, N_γ, N_q and N_c are functions of ϕ and, to some extent, the depth of penetration. Table II gives values appropriate to depths up to one footpad diameter.

Equation 10 represents the superposition of three special-case solutions, and is not an exact solution of the general case. The first two terms assume a non-cohesive soil deriving its shear strength only from friction. The first term gives the frictional bearing strength at zero penetration resulting from the normal stresses generated by the weight of the soil and the force applied by the footpad. It reflects the well recognized fact that the surface bearing strength of cohesionless soils, like sand, increases linearly with the size of the footpad.

The second term is derived from a solution for the frictional bearing strength that would be caused by a uniform surface loading or surcharge around the footpad. If the footpad is on the surface, there is no surcharge and this term is zero. If the footpad has penetrated, the weight of soil above the level of the footpad can be considered as applying a surcharge to the soil below this level. Substituting the weight per unit area of the soil above the footpad level for the surcharge in this solution results in a term which gives the increase in static bearing strength with depth. For a zero-strength soil, $\phi = c = 0$, this term reduces to a buoyancy force.

The third term represents the contribution of cohesion when both the soil weight and surcharge are considered zero. This is the inherent strength of the soil, independent of external forces or gravity effects.

To calculate the dynamic augmentation for the incompressible soil, a dynamic model of the soil flow is needed to insure that all the dynamic forces are accounted for correctly. This model, shown in Figure 3, indicates that the four masses of moving soil are linked together at definite velocity ratios, as if by levers, by the requirement for incompressible flow. The lever ratios shown on Figure 3 are the same as the speed ratios, and are derived from the soil flow model, Figure 2. Soil mass #1 has no lever arm because it moves at the same velocity as the penetrating footpad; soil mass #4 has the longest lever arm because it moves at only one-eighth the footpad velocity. The other two soil masses move at intermediate speeds. The dynamic force of soil mass #1 is transmitted directly to the footpad; the dynamic forces of soil masses #2, #3 and #4 are transmitted to the footpad through the lever system.

The dynamic force per unit footpad area of each soil mass is equal to the time rate of change of momentum of that soil mass divided by the footpad area, A. The total dynamic augmentation resisting the penetration of the footpad is the sum of these individual dynamic forces each multiplied by the appropriate lever ratio.

$$P_A = \frac{d}{dt}\left(\frac{M_1}{A}v_1\right) + \frac{1}{2.83}\frac{d}{dt}\left(\frac{M_2}{A}v_2\right) + \frac{1}{6.12}\frac{d}{dt}\left(\frac{M_3}{A}v_3\right)$$

$$\frac{1}{8}\frac{d}{dt}\left(\frac{M_4}{A}v_4\right) \qquad (11)$$

Substituting the mass per unit area and velocity values from Table I gives:

$$P_A = \frac{d}{dt}\left(\frac{\rho r}{3}\dot{y}\right) + \frac{1}{2.83}\frac{d}{dt}\left(\pi\rho r\frac{\dot{y}}{2.83}\right) + \frac{1}{6.12}\frac{d}{dt}\left(4\rho r\frac{\dot{y}}{6.12}\right) +$$

$$\frac{1}{8}\frac{d}{dt}\left(9\rho y\frac{\dot{y}}{8}\right) \qquad (12)$$

The lever ratios appear twice in equation 12, once because the speeds of the soil masses are less than the footpad speed and once because the effectiveness of the individual dynamic forces is reduced by the levers. Simplifying and adding the static bearing strength, the dynamic bearing strength of incompressible soil becomes:

$$P_{DYN} = 0.6\,\rho g r N_\gamma + \rho g y N_q + 1.3\,c N_c + 0.14\,\rho\dot{y}^2 +$$

$$(0.83\,r + 0.14\,y)\rho\,\ddot{y} \qquad (13)$$

GENERALIZED DYNAMIC BEARING STRENGTH

When calculating the penetration of a spacecraft footpad into the soil of an extraterrestrial body, the soil properties and mode of failure will generally both be unknown. To investigate the full range of possible landing conditions, the values of ρ_1, C, p_o and b will have to be varied over reasonable ranges for compressible soils and then the values of ρ, c and ϕ must be varied for the incompressible soils. A generalized dynamic bearing strength formula applicable to both compressible and incompressible soils would reduce the amount of calculations necessary. It would also provide for the possibility of soil failure by a combination of the two modes provided that additional terms are not introduced into the dynamic bearing strength expression when the soil fails in a combination of the two modes. Such a formula can be written by generalizing the expression for static bearing strength and introducing the concept of effective soil mass.

The static bearing strength of both the compressible and incompressible soils can be reasonably approximated by linear functions of the penetration depth (equations 6 and 10). This suggests using a general linear relationship to represent the static bearing strength of the soil regardless of the failure mode.

$$P_s = A + By \tag{14}$$

If the soil is compressible, equation 6 applies:

$$A = p_o$$

$$B = \frac{b - \rho_1 g}{C}$$

If the soil is incompressible, equation 10 applies:

$$A = 0.6\ \rho g r N_\gamma + 1.3\ c N_c$$

$$B = \rho g N_q$$

Effective soil mass, m_e, is the imaginary mass per unit penetrated area which moves at the penetration velocity remaining in contact with the penetrating object and exerting the same dynamic augmentation pressure as the real soil masses. The dynamic augmentation is the time rate of change of momentum of the effective soil mass.

$$P_A = \frac{d}{dt}\ (m_e\ \dot{y}) \tag{15}$$

The dynamic bearing strength is the sum of the static bearing strength and the dynamic augmentation.

$$P_{DYN} = P_s + \frac{d}{dt}(m_e \dot{y}) \tag{16}$$

In compressible soil the dynamic augmentation is produced by the compressed slug of soil moving at the penetration velocity. The effective mass is therefore the mass of this slug of soil divided by the penetration area. This is shown by combining equations 4, 7 and 15.

$$P_A = \frac{d}{dt}(m_e \dot{y}) = \frac{d}{dt}\left(\frac{M}{A}\dot{y}\right)$$

$$m_e = \frac{M}{A} = \frac{\rho_1}{C} y \tag{17}$$

In incompressible soil the dynamic augmentation is produced by four soil masses only one of which moves at the penetration velocity. The soil mass #1 is entirely effective; however, the other masses are less effective because they do not move at the full penetration velocity. The effective mass of this system can be found by combining equations 12 and 15.

$$P_A = \frac{d}{dt}(m_e \dot{y}) = \frac{d}{dt}\left(\frac{\rho r}{3}\dot{y}\right) + \frac{1}{2.83}\frac{d}{dt}\left(\pi\rho r \frac{\dot{y}}{2.83}\right) +$$

$$\frac{1}{6.12}\frac{d}{dt}\left(4\rho r \frac{\dot{y}}{6.12}\right) + \frac{1}{8}\frac{d}{dt}\left(9\rho y \frac{\dot{y}}{8}\right)$$

$$m_e = \frac{\rho r}{3} + \left(\frac{1}{2.83}\right)^2 \pi\rho r + \left(\frac{1}{6.12}\right)^2 4\rho r + \left(\frac{1}{8}\right)^2 9\rho y$$

The effective mass is the sum of the individual masses multiplied by the square of their velocity ratios relative to the penetration velocity. Simplifying the expression:

$$m_e = 0.83\rho r + 0.14\rho y \tag{18}$$

The effective mass of both the compressible and incompressible soils can be represented by a linear function of the penetration depth:

$$m_e = E + Fy$$

E is the effective soil mass in contact with the object at the surface, F is the rate at which effective mass is accumulated with depth. If the soil is compressible, equation 17 applies:

$$E = 0$$

$$F = \frac{\rho_1}{C}$$

If the soil is incompressible, equation 18 applies:

$$E = 0.83\rho r$$

$$F = 0.14\rho$$

The generalized formula for dynamic bearing strength is found by substituting the generalized equations for static bearing strength and effective mass into equation 16 and expanding the derivative.

$$P_{DYN} = A + By + (E + Fy)\ddot{y} + F\dot{y}^2$$

The generalized soil parameters A, B, E and F not only allow a single dynamic bearing strength formula to be used for both compressible and incompressible soils, they also provide a convenient means of describing a soil in terms meaningful to a spacecraft designer who is primarily interested in the dynamic bearing strength. A is the surface static bearing strength. B is the rate at which static bearing strength increases with depth. E is the mass of soil which is, in effect, stuck to each unit area of the penetrating object as it penetrates the surface. F is the rate of accumulation of this mass with depth. Given these four parameters, the designer can calculate the dynamic reaction forces for normal penetrations.

The generalized dynamic bearing strength formula assumes that no new terms are introduced when the soil fails in a combination of the incompressible and compressible modes. It is also limited by all the assumptions made in the separate analysis of the dynamic bearing strength of compressible and incompressible soils, particularly: that the dynamic soil motion is the same as the static soil motion and that the incompressible soil dynamic terms are not significantly affected by friction.

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Attachments
Figures 1-3
Tables I, II

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REFERENCES

1. Scott, Ronald F., Problem of the Penetration of a Projectile into Soil, a Soil-Like Medium, or Compressible Rock (Pumice), Unpublished, Report to Space General, Inc., October 26 through November 20, 1962.

2. Scott, Ronald F., Principles of Soil Mechanics, Addison-Wesley Publishing Company, Inc., 1963.

3. Terzaghi, Karl and Peck, Ralph B., Soil Mechanics in Engineering Practice, John Wiley and Sons, Inc., 1962.

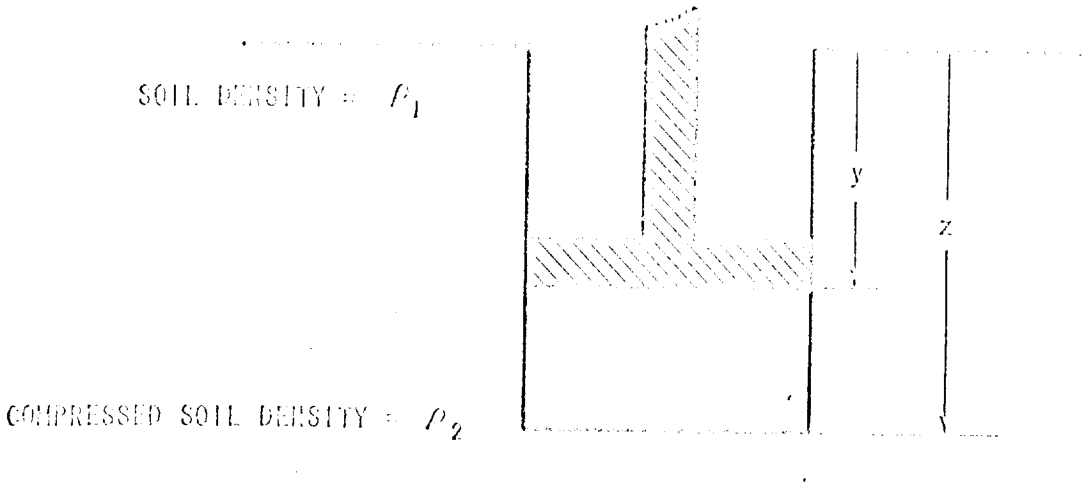

FIGURE 1 - SOIL FLOW MODEL FOR COMPRESSIBLE SOIL

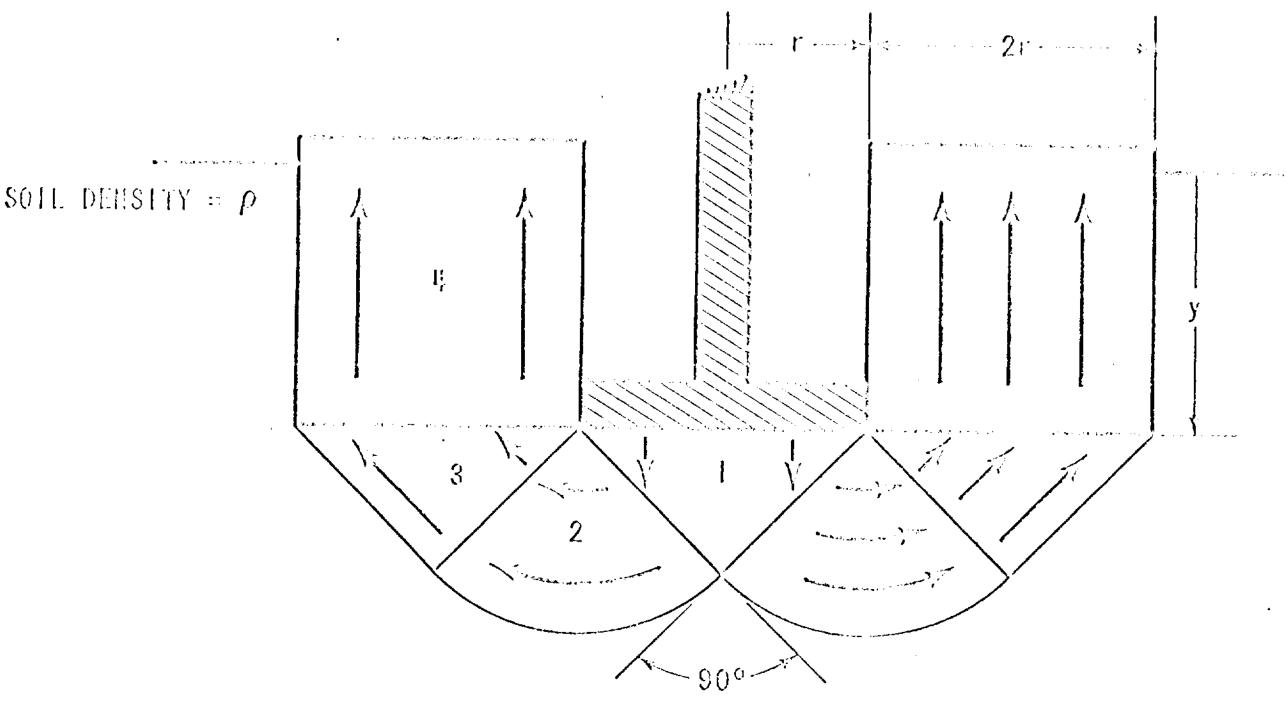

FIGURE 2 - SOIL FLOW MODEL FOR INCOMPRESSIBLE SOIL

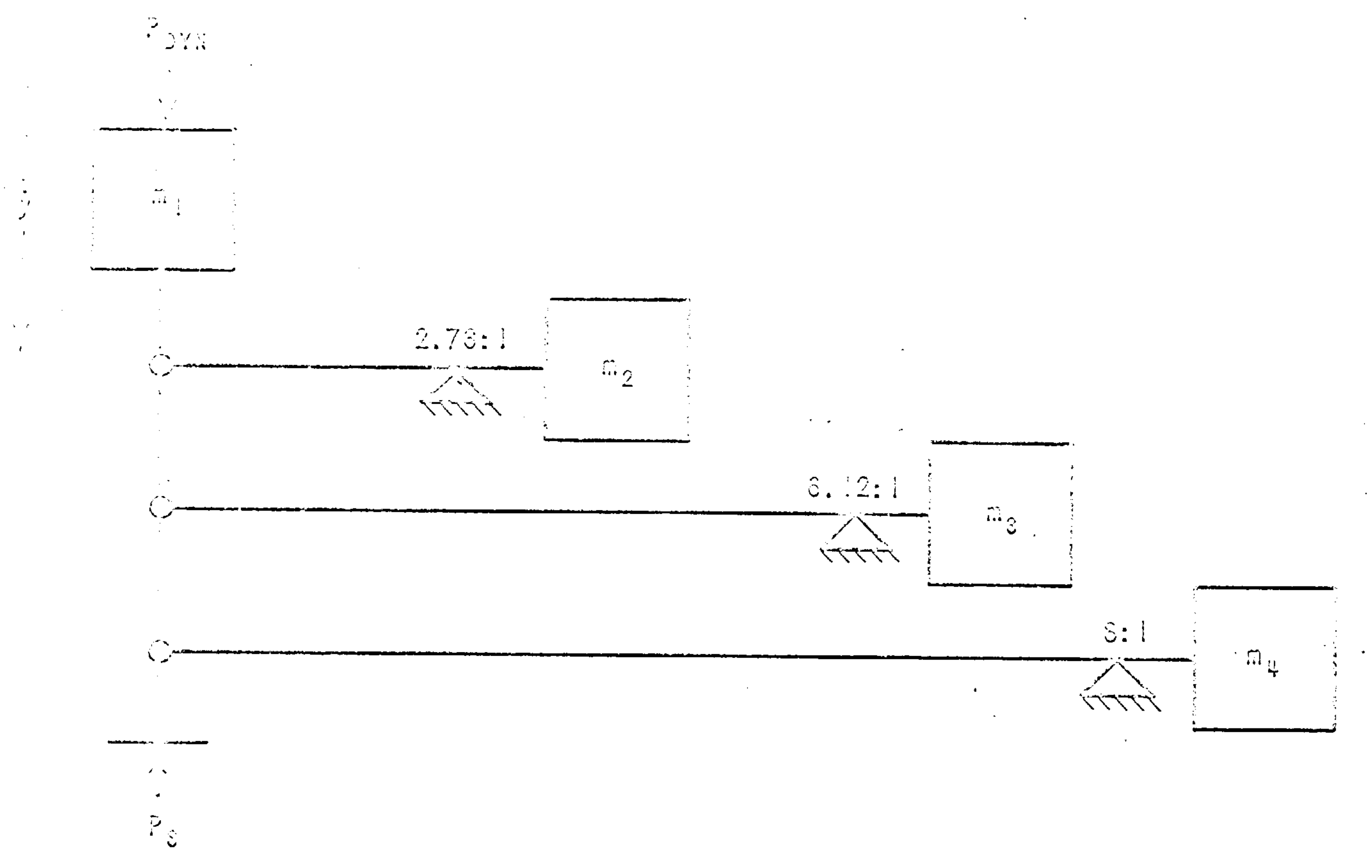

FIGURE 3 - DYNAMIC MODEL FOR INCOMPRESSIBLE SOIL

SOIL VOLUME	MASS PER UNIT AREA	AVERAGE VELOCITY
#1	$\frac{\rho r}{3}$	$\dot{y}$
#2	$\pi \rho r$	$\frac{1}{2.83}\dot{y}$
#3	$4\rho r$	$\frac{1}{6.12}\dot{y}$
#4	$9\rho y$	$\frac{1}{8}\dot{y}$

TABLE I

MASSES AND VELOCITIES OF DISPLACED INCOMPRESSIBLE SOIL VOLUMES

ϕ	0°	10°	20°	30°	40°
N_γ	0	0.8	5.5	26	100
N_q	1	2.8	8	25	85
N_c	5	9	16	40	100

TABLE II

BEARING CAPACITY FACTORS FOR INCOMPRESSIBLE SOILS